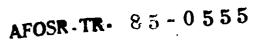


NATIONAL BUREAU OF STANDARDS MICROCOPY RESOLUTION TEST CHART





Title

APPLICATIONS OF DIFFERENTIAL PHASE STATISTICS TO STUDIES OF C3 AND SPREAD SPECTRUM COMMUNICATIONS

Grant

F49620-83-C-0085

Final

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MATTHEW J. KERVER
FINAL REPORTER, Technical Information Division

APPLICATIONS OF DIFFERENTIAL PHASE STATISTICS TO STUDIES OF C<sup>3</sup> AND SPREAD SPECTRUM COMMUNICATIONS

### (a) Statement of Work

- (i) Spread spectrum modulation techniques. Investigate the bit error rate performance of spread spectrum and  $\mathbb{C}^3$  systems employing differential modulation when operating under conditions of time and frequency errors, and jamming in the M-ary case. Investigate the use of adaptive antijam techniques to combat jamming in these situations.
- (ii) New mathematical approaches. Develop new mathematical techniques, such as the use and applicability of asymptotic analyses, as needed to successfully attain the above goals.
- (iii) Results. Compare results with previous analytical work of other researchers. Prepare reports so that they will be suitable for submission to scholarly technical journals.

### (b) Status of Research Effort

Three reports have been written under this second year of the contract and are listed in the next section as Papers No. 6, 7, and 8. Paper No. 6 addresses the classical and very complicated problem of finding the probability density function at the output of an RC filter when the input is a binary random telegraph signal whose intervals are described by a renewal process. This paper makes great headway in solving the problem and represents the first new results in this area since McFadden's original paper published in 1959 in the IEEE Transactions on Information Theory. Some important new closed form and series results are obtained in the case when the interval density is Gamma distributed. It is shown how the problem can be cast in the equivalent forms of integral and differential equations, and iterative and matrix approximation techniques are developed for solving these equations. Comparisons of the methods are made with approximations based upon the Fokker-Planck equation. These studies represent part of our continuing efforts to develop new mathematical approaches and techniques which might ultimately have application in assessing system performance in certain practical cases. Probability density functions of these types have been used by others as models of the phase noise in certain phase-lock loops.

Paper No. 7 focuses on a particular case in which the desired probability density is the solution of a rather formidable third-order differential equation with non-constant coefficients. A closed form solution is found for one special value of a system characteristic parameter, and a series solution is obtained for general values of this parameter. In one of the solutions, it is necessary to evaluate the modulus of a gamma function with a complex argument, and a method is developed for doing the calculation by means of a simple numerical integration. The results in Paper No. 7 were incorporated into Paper No. 6 and the revised Paper No. 6 has since been accepted for publication in the <a href="IEEE Transactions on Information Theory">IEEE Transactions on Information Theory</a>.

Paper No. 8 is concerned with developing methods for calculating the solution to a fourth order differential equation. Although the theory of such equations is well known, a prohibitive amount of algebra is required to determine the coefficients in series solutions, and a computer method is developed in which the algebra is circumvented. The method is quite general, and in an appendix is applied to Bessel's differential equation and, quite surprisingly, a new second solution to Bessel's equation is obtained which does not appear to have been previously noticed. This new second solution has the advantage that its series expansion is simpler than the classical second solution given in Watson's book on Bessel functions. This paper has been submitted for publication in the IEEE Transactions on Information Theory.

We are progressing in our studies of the effects of time and frequency errors and jamming in the M-ary case of spread spectrum systems which use differential modulations. In the case when the communications system employs an active jammer state monitor, we have developed symbol error rate probabilities for the cases of M-ary PSK and M-ary DPSK. These results clearly demonstrate how the error rate is changed from an exponential dependence upon the transmitted energy-to-noise ratio to an inverse power dependence. Such formulas are important and necessary for the optimization of system performance when there is a trade-off between anit-jam performance and anti-intercept performance when covertness is desired. The differential phase statistics developed in part by the Principal Investigator play an important role in these analyses.

(c) List of Reports and Written Publications in Technical Journals.

Following is a list of all papers and reports supported under this contract since its beginning:

- 1. R.F. Pawula and J.H. Roberts, "The Effects of Noise Correlation and Power Imbalance on Terrestrial and Satellite DPSK Channels," IEEE Transactions on Communications, COM-31, pp. 750-755, June 1983.
- R.F. Pawula, "Offset DPSK and a Comparison of Conventional and Symmetric DPSK with Noise Correlation and Power Imbalance, Proceedings of MILCOM '83, Washington, D.C., pp. 93-98, October 1983.
- 3. R. F. Pawula, "Asymptotics and Error Rate Bounds for M-ary DPSK," IEEE Transactions on Communications, COM-32, pp. 93-94, Jan. 1984.
- 4. R.F. Pawula, "Offset DPSK and a Comparison of Conventional and Symmetric DPSK with Noise Correlation and Power Imbalance," IEEE Transactions on Communications, COM-32, pp. 233-240, March 1984.
- 5. R.F. Pawula, "On M-ary DPSK Transmission Over Terrestrial and Satellite Channels," <u>IEEE Transactions on Communications</u>, <u>COM-32</u>, pp. 752-761, July 1984.
- 6. R.F. Pawula and S.O. Rice, "On Filtered Binary Processes," accepted for publication in the <u>IEEE Transactions on Information Theory</u>.
- 7. R.F. Pawula, "On Filtered Binary Processes with  $f(t)=a^2t \exp(-at)$ ," Random Applications, Inc., Montrose, CO, Jan. 1985.
- 8. R.F. Pawula and S.O. Rice, "A Differential Equation Related to a Random Telegraph Wave Problem Computer Calculation of Series Solution," submitted to the IEEE Transactions on Information Theory.

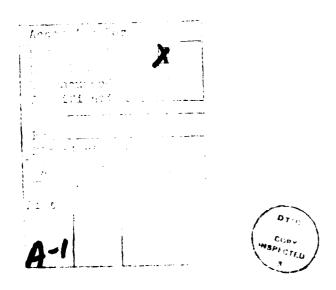
Paper No. 2 is an abridged version of Paper No. 4. The results of Paper No. 7 have been incorporated into a revised version of Paper No. 6.

## (d) List of Professional Personnel Associated with Research Effort

All of the work done under this contract was done by the Principal Investigator, Dr. Robert F. Pawula. Papers No. 6 and 8 were written with the collaboration of Dr. S.O. Rice of the University of California at San Diego. Dr. Rice and the Principal Investigator have been collaborating over the past fifteen years and have written four papers together with the first of these being published in 1973 in the IEEE Transactions on Information Theory. The various mathematical aspects of our work are routinely discussed with Mr. John H. Roberts of the Plessey Electronics Company, Roke Manor, Romsey, Hampshire, England. Mr. Roberts put us into contact with Professor A. Munford who is with the Department of Mathematical Statistics and Operations Research at the University of Exeter, Exeter, England. Prof. Munford's work on moments was inspired by our correspondence and is referenced in Paper No. 6. We also have been in close contact with Professor I. Korn in the Electrical Engineering Department at the University of New South Wales in Australia concerning our work on digital frequency modulation and the use of our differential phase statistics.

# (c) Interactions (Coupling Activities)

- (i) Attendance at Meetings. The Principal Investigator attended the 1984 IEEE Military Communications Conference in Los Angeles, CA.
- (ii) Consultative functions to other agencies. The Principal Investigator consults on an on-going and informal basis with Dr. Marvin K. Simon of the Jet Propulsion Laboratory of NASA on problems dealing with the mathematical aspects of differential modulations. The Principal Investigator has reviewed papers for the IEEE Transactions on Communications and the IEEE Transactions on Information Theory during the course of this contract.



# END

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